b) En déduire que $\int_0^{\alpha} f(x)\,dx = \dfrac{\alpha^2-\alpha+1}{\alpha-1}$.

$$\int_0^{\alpha} f(u)\,du = \int_0^{\alpha} 1 + \frac{\overbrace{2u}^{u'}}{\underbrace{2\sqrt{u^2+1}}_{2\sqrt{u}}}\,du$$

$\dfrac{u'}{2\sqrt{u}} \xrightarrow{P} \sqrt{u}$

$$= \Big[u + \sqrt{u^2+1}\Big]_0^{\alpha}$$

$$= \alpha + \sqrt{\alpha^2+1} - 1$$

$$= \alpha - 1 + \sqrt{\alpha^2+1}$$

$$= \alpha - 1 + \frac{\alpha}{\alpha-1}$$

$$= \frac{(\alpha-1)(\alpha-1)+\alpha}{\alpha-1}$$

$$= \frac{\alpha^2-2\alpha+1+\alpha}{\alpha-1}$$

$$\int_0^{\alpha} f(x)\,du = \frac{\alpha^2-\alpha+1}{\alpha-1}$$